

# Myopia and Pensions in General Equilibrium\*

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## Abstract

The US social security tax rate has doubled in the last half century. Does the degree of myopic behavior that we observe in the US justify the size of the social security program? To study this question we build a computable general equilibrium model that is composed of life-cycle permanent-income consumers who save optimally and “hand-to-mouth” consumers who just consume their disposable income. Our model is a continuous-time, general equilibrium extension of the model by Cremer, De Donder, Maldonado, and Pestieau (2008), though we abstract from the redistributive function of social security to focus on myopia. Retirement is a choice variable in our model and the social security program is designed to mimic the US program in which the annuity value of benefits increases with the retirement age. Also, we allow for delayed claiming beyond the date of retirement. The model matches a variety of important data targets relating to saving and retirement. We find that small reductions in the social security tax rate provide significant welfare gains to *both groups* of consumers.

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# 1 Introduction

The OASI social security tax rate in the US has doubled in the last half century, and it is five times larger than when the program was created in 1935. Currently, the OASI part of social security is funded by a 10.6% payroll tax (split evenly between employers and employees), making it the largest government program in the world. Social security is traditionally justified among the lay public on the grounds that people are myopic and have trouble saving for retirement on their own. If we take this foundation seriously in modeling the program, has it grown too big? That is, does the degree of myopic behavior that we observe in the US justify the size of the social security program?<sup>1</sup> Our focus on the US is merely for convenience since pay-as-you-go programs with similar institutional features are common throughout the OECD.

To study this question we build a general equilibrium life-cycle model with overlapping generations of individuals who choose when to retire. Our economy is composed of a mixture of life-cycle permanent-income (LCPI) consumers who save optimally and “hand-to-mouth” consumers who don’t save at all. The LCPI consumers solve a two-stage optimal control problem that defines optimal consumption, saving, and retirement age for a given set of factor prices, social security taxes, and social security benefit incentives. The hand-to-mouth consumers just consume their disposable income and never save for the future, but they do choose when to retire. At the macro level, factor prices are determined competitively and the social security administration runs a pay-as-you-go program with a balanced budget. Lifetime utility in a stationary competitive equilibrium is the welfare criterion for both consumers, even though the hand-to-mouth consumers do not maximize utility. This paternalistic approach opens up the possibility that social security could be welfare improving.

Quantitatively speaking, our model performs fairly well along some dimensions that are important for studying the question that we have posed. For example, the model endogenously produces a realistic capital-output ratio and a realistic interest rate, realistic retirement choices, a realistic discrete drop in consumption spending at the date of retirement, and a realistic incentive structure that rewards delayed collection of social security benefits. The model also bears a realistic portion of the population who never save for retirement (20% in the US), so it appears to be a reasonable instrument for quantitatively studying myopia as a justification for social security.

We compare lifetime utilities in a stationary competitive equilibrium with the current tax rate to lifetime utilities in a different equilibrium with a smaller tax rate. We find that both groups of consumers prefer the equilibrium with the smaller social security tax. Indeed, a small reduction in the tax rate can generate fairly large welfare gains, measured as the percentage increase in consumption in each period that is necessary to achieve the level of lifetime utility in an equilibrium with a smaller tax rate. This “compensating variation” is similar for both groups of consumers and it increases nearly linearly with the size of the tax cut. A reduction in the social security tax rate from 10.6% to 9% leads to compensating variations for both groups of consumers that are above 1%. A reduction in the tax rate

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<sup>1</sup>As in similar studies, we assume myopia is the primary rationale for the existence of a social security program. We abstract from the other roles that social security programs play in insuring against longevity risk and redistributing income among the elderly.

that is twice as large leads to compensating variations that are above 2%, etc.<sup>2</sup> But, we emphasize that these results do not necessarily mean the US program is too big. We remain agnostic about that. Instead, in the context of the model, the results mean the social security program is bigger than can be justified by the level of myopia present in the US, leaving other roles such as redistribution of income as a necessary part of the rationale for social security.

To our knowledge, our paper is the first general equilibrium study of the optimal provision of social security when the economy is a mixture of LCPI and hand-to-mouth individuals and when retirement is endogenous. Feldstein (1985), Docquier (2002), Hurst and Willen (2007), and Cremer, De Donder, Maldonado, and Pestieau (2008) study the optimal design of pay-as-you-go social security for a mixed economy in partial equilibrium. Caliendo and Gahramanov (2009) study the optimal design of social security for a mixed economy in general equilibrium, but retirement is exogenous. Therefore, the optimal rate of social security taxation is an open question, for a general equilibrium, mixed economy with endogenous retirement.

Our model is a continuous-time, general equilibrium extension of the two-period, partial equilibrium model created by Cremer, De Donder, Maldonado, and Pestieau (2008).<sup>3</sup> The purpose of their paper is to derive the optimal level of social security taxation *and* the optimal degree of redistribution (from elderly wealthy to elderly poor) when the economy is composed of both LCPI and hand-to-mouth consumers who vary by labor productivity. They primarily make an analytical contribution, while we try to make a quantitative point with a computable general equilibrium model. We abstract from intra-cohort heterogeneity in labor productivity to focus on the mandatory saving role of social security.<sup>4</sup> More broadly speaking, our work falls into a large, normative literature that studies the optimal provision of social security under myopia, pioneered by Feldstein (1985) and revived by İmrohoroğlu, İmrohoroğlu, and Joines (2003).<sup>5</sup>

It is useful to study this question in general equilibrium for a number of reasons. In actuality, social security affects life-cycle consumption and saving decisions and it affects

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<sup>2</sup>Cremer and Pestieau’s (2000) position is that “majority voting typically results in overspending on social security.”

<sup>3</sup>Additionally, the way we handle endogenous retirement follows Cremer, Lozachmeur, and Pestieau (2004).

<sup>4</sup>Technically speaking, we are not able to *completely* abstract from redistribution, even though there are no differences in productivity. The agents of differing types choose different dates of retirement, so they bear different portions of the tax burden. The fact that benefits increase with delayed retirement in the model removes much of the redistribution. However, calibrating to the US social security age-benefit rule does not guarantee perfect elimination of this sort of redistribution. Thus, the direction of the redistribution (whether from LCPI to hand-to-mouth or the other way) depends on the general equilibrium choices of the consumers and the parameterization of the benefit rule.

<sup>5</sup>This literature is too large to cite properly here, but see the many references in Findley and Caliendo (2009) and Caliendo and Gahramanov (2009) for background, and also see the surveys by Findley and Caliendo (2008) and Cremer and Pestieau (2009). The many papers from this literature systematically examine whether various forms of myopia, such as standard discounting (Feldstein (1985), Andersen and Bhattacharya (2008), Cremer, De Donder, Maldonado, and Pestieau (2009)), hyperbolic discounting (İmrohoroğlu, İmrohoroğlu, and Joines (2003), Caliendo (2009)), short planning horizons (Findley and Caliendo (2009)), temptation preferences (Buccioli (2006), Buccioli (2008), Kumru and Thanopoulos (2008)), and limited computational ability (Caliendo and Findley (2009)) are able to rationalize a program as large as the one in the US. The present assumption that a fraction of the population live hand-to-mouth is the most extreme description of myopia and therefore should bias the results in favor of a large social security program.

retirement choices. These distortions affect the supply of aggregate capital and labor and hence factor prices. Also, the social security budget comes from the tax base, which is affected by retirement choices and factor prices. A general equilibrium model is able to account for these distortions by assuring that, whatever households choose to do at the micro level in response to social security, at the macro level factor markets must clear and the social security budget must balance. The drawback is that we have fewer analytical results since our analysis is largely computational, but the fundamental micro-level intuition is already well understood from papers like Cremer, De Donder, Maldonado, and Pestieau (2008).<sup>6</sup>

Our work is also closely related to a large, positive literature that studies the effect of various institutional features of social security programs on labor force participation in old age (recent examples include Diamond and Gruber (1997), Gruber and Wise (1998), Samwick (1998), Börsch-Supan (2000), Coile, Diamond, Gruber, and Jousten (2002), Cremer and Pestieau (2003), Gustman and Steinmeier (2005), Coile and Gruber (2007), Cigno (2008), Liebman, Luttmmer, and Seif (2009), Mastrobuoni (2009), and Ortiz (2009) to name just a few). For example, Diamond and Gruber (1997) have done a thorough analysis of the incentives associated with initiation of benefits at different ages by calculating how social security wealth evolves as a function of retirement age. Our contribution is to extend their work by embedding these incentives in a computable general equilibrium model and then to quantify the welfare consequences of reductions in the size of the social security tax rate.

Finally, we pay special attention to the issue of whether it is optimal to delay collection of benefits beyond the date of retirement. In the US, collection of social security benefits and retirement are not required to be synchronized. One doesn't actually need to work longer to qualify for a larger annuity, one can retire early and just postpone collection. Coile, Diamond, Gruber, and Jousten (2002) point out that allowing for this institutional feature can prevent the model from generating biased predictions. We attempt to build the type of model that they have put on the research agenda:

*“A full analysis of the problem of delayed claiming would model jointly the retirement and the claiming decision. Such a model is beyond the scope of the current effort.”*

We make progress along this front by augmenting our optimal control problem of endogenous retirement to also allow for delayed claiming. The problem becomes a complicated

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<sup>6</sup>We direct readers to Andersen and Bhattacharya (2008) for a careful theoretical treatment of the limits of myopia as a rationale for social security. In particular, if myopia takes the form of a discount rate used in decision making that is less than the one used in welfare calculations, then myopia must be paired with another friction (such as pessimism or borrowing constraints) or must be studied in general equilibrium in order to rationalize social security in a dynamically efficient economy. Basically, Andersen and Bhattacharya show that if the analysis is in partial equilibrium, then myopia needs to be severe enough that the individual is observationally hand-to-mouth (meaning the borrowing constraint always binds) or there must be enough pessimism (as in Feldstein (1985)). In general equilibrium, if social security reduces capital accumulation and hence raises the interest rate, a forward looking consumer will choose a steeper consumption profile with social security than without, and the steeper profile is more in line with the profile that maximizes true utility. Hence, Andersen and Bhattacharya make it clear that both hand-to-mouth behavior and general equilibrium effects can act as channels to help rationalize social security. Both of these channels are present in our study.

three-stage optimal control problem with two endogenous switch points. The agent jointly optimizes along the consumption, retirement, and benefit collection dimensions. We show that our welfare results are robust, in general equilibrium, to the inclusion of this institutional feature.

## 2 Microfoundations

Since LCPI consumers, by definition, do not need help saving for retirement, mandatory participation in a program with a below market internal rate of return is not helpful (which occurs when the economy is dynamically efficient). We follow Feldstein (1985), Docquier (2002), Hurst and Willen (2007), Cremer, De Donder, Maldonado, and Pestieau (2008) and others, and allow a portion of the population to deviate from the LCPI rule in the extreme and just consume all of their income without any type of planning. This opens up the possibility that social security can provide old age support for a fraction of the population.

### 2.1 LCPI Consumers

Time is continuous and is indexed by  $t$ . The individual starts work at  $t = 0$ , retires by choice at  $t = T$  and then passes away and exits the model at  $t = \bar{T}$ . Retirement is a choice but is irreversible. The date of death is certain and fixed.

The individual's period utility function over consumption  $c(t)$  and leisure  $l(t)$  takes a separable form that is standard in similar macroeconomic models (Heijdra and Ligthart (2006), Ortiz (2009)):  $u[c(t), l(t)] = \sigma \ln c(t) + (1 - \sigma) \ln l(t)$ , where  $\sigma$  is a parameter that captures the trade-off between consumption and leisure in the period utility function.

To simplify and focus on the endogenous retirement aspect of the problem rather than the continuous leisure choice over the worklife, we model leisure as a binary choice following Samwick (1998), Cremer and Pestieau (2003), Cremer, Lozachmeur, and Pestieau (2004), Ortiz (2009) and others. Let  $l(t)$  take on a low value  $l_-$  during the working period and a high value  $l_+$  during retirement. We let the individual decide when to switch from  $l_-$  to  $l_+$ . We normalize  $l_+ = 1$ . Hence  $l_-$  has the interpretation of the fraction of available time during the working period that is spent in leisure.

The individual is endowed with  $e(t)$  labor efficiency units at time  $t$ , for which he is paid a wage  $w$  per efficiency unit supplied to the labor market. He must pay a social security tax rate  $\theta$  on wage income during the working period. During retirement, he receives a constant flow of social security benefits  $b(T)$  that depends on the date of retirement (as in, e.g., Samwick (1998) and others). For now, we follow convention and assume that retirement and collection of benefits is a single, synchronized choice. Later in the paper we allow the individual to delay collection of benefits beyond the date of retirement. Any after-tax income that is not consumed is deposited in a savings account  $k(t)$ , which earns interest at rate  $r$ . The individual starts and finishes the life cycle with nothing in his savings account,  $k(0) = k(\bar{T}) = 0$ .

The individual solves

$$\max_{\{c(t)\}_T} \int_0^T \exp[-\rho t] [\sigma \ln c(t) + (1 - \sigma) \ln l_-] dt + \int_T^{\bar{T}} \exp[-\rho t] \sigma \ln c(t) dt, \quad (1)$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + (1 - \theta)(1 - l_-)e(t)w - c(t), \text{ for } t \in [0, T], \quad (2)$$

$$\frac{dk(t)}{dt} = rk(t) + b(T) - c(t), \text{ for } t \in [T, \bar{T}], \quad (3)$$

$$k(0) = k(\bar{T}) = 0, \quad (4)$$

$$b(T) = z_0 + \frac{z_1}{1 + \exp[z_2 - z_3 \times T]}, \quad z_0, z_1, z_2, z_3 \in \mathbb{R}^+, \quad (5)$$

where  $\rho \in \mathbb{R}^+$  is the discount rate. Note that we have made use of the fact that  $\ln l^+ = 0$ , and we are using a logistic function to approximate the real-world benefit formula described earlier where  $z_1$ ,  $z_2$ , and  $z_3$  are exogenous parameters and  $z_0$  is a scale parameter that will be determined endogenously in general equilibrium to balance the budget. Also note that  $b'(T) > 0$ .<sup>7</sup> The individual takes factor prices  $w$  and  $r$  as fixed and takes social security parameters  $\theta$ ,  $z_0$ ,  $z_1$ ,  $z_2$ , and  $z_3$  as fixed and then chooses consumption, saving, and retirement optimally.

This is a two-stage optimal control problem with an endogenous switch point and a fixed endpoint on the state variable (Tomiya (1985)). The problem is “two-stage” because the objective functional and state equation both involve discrete switches. The switch point  $T$  is endogenous because retirement is a choice variable in this problem. We can solve this control problem by first recognizing that, for any  $T$ , the consumption path must be chosen optimally. Hence, we proceed as follows: (i) solve a two-stage control subproblem where  $T$  is exogenous, and (ii) then solve for the optimal switch point given that consumption is chosen optimally for every possible switch point, i.e., search across all the solutions from the subproblems in the previous step for the one that maximizes utility.

The optimal consumption profile, from the two-stage control subproblem with an exogenous retirement date  $T$ , is

$$c_T(t) = \frac{\int_0^T (1 - \theta)(1 - l_-)e(s)w \exp[-rs] ds + \int_T^{\bar{T}} b(T) \exp[-rs] ds}{\exp[-(r - \rho)t] \int_0^{\bar{T}} \exp[-\rho s] ds}, \quad (6)$$

for  $t \in [0, \bar{T}]$ . Next, we endogenize the date of retirement by maximizing the objective function with respect to  $T$ , setting consumption equal to its subproblem solution from (6)

$$\begin{aligned} T^* &= \arg \max \left[ \int_0^T \exp[-\rho t] [\sigma \ln c_T(t) + (1 - \sigma) \ln l_-] dt + \int_T^{\bar{T}} \exp[-\rho t] \sigma \ln c_T(t) dt \right] \\ &= \arg \max \left[ \int_0^T \exp[-\rho t] (1 - \sigma) \ln l_- dt + \int_0^{\bar{T}} \exp[-\rho t] \sigma \ln c_T(t) dt \right]. \end{aligned} \quad (7)$$

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<sup>7</sup>In addition to the US, many countries in the OECD have social security programs in which  $b'(T) > 0$ . Cremer, Lozachmeur, and Pestieau (2004) ask the normative question of whether this is a good way to design a pension system. It is the redistributive feature of the program that may be welfare improving in their paper, since they abstract from myopia. Also see Cremer and Pestieau (2003) and Casamatta, Cremer, and Pestieau (2005).

The second step in (7) is due to the fact that  $c_T(t)$  from (6) is a smooth function that holds for the entire lifespan  $t \in [0, \bar{T}]$ .

## 2.2 Hand-to-Mouth Consumers

By assumption, a hand-to-mouth consumer does not save, but he does choose retirement (total leisure time)

$$\max_T \int_0^T \exp[-\rho t] [\sigma \ln c(t) + (1 - \sigma) \ln l_-] dt + \int_T^{\bar{T}} \exp[-\rho t] \sigma \ln c(t) dt, \quad (8)$$

subject to

$$c(t) = (1 - \theta)(1 - l_-)e(t)w, \text{ for } t \in [0, T], \quad (9)$$

$$c(t) = b(T), \text{ for } t \in [T, \bar{T}]. \quad (10)$$

This is the continuous time analogue to the hand-to-mouth consumer in Cremer, De Donder, Maldonado, and Pestieau's (2007, 2008) two-period model, in which the myopic individuals do not save anything for retirement but they do choose their leisure time according to an optimization problem. Leisure choice in the first period of a two-period model can be interpreted as endogenous retirement (also see the companion paper Cremer, De Donder, Maldonado, and Pestieau (2009)).

The hand-to-mouth consumer faces a trade-off. If he continues to stay in the labor force he will continue to receive wage income, but he foregoes leisure time and social security benefits. The problem is complicated by the fact that his wage income is dynamic due to the evolution of his efficiency endowment with age and the social security annuity that he receives is sensitive to the date of retirement. Also, working until death is a possibility in theory, provided his efficiency endowment never drops all the way to zero, which would reflect complete disability.<sup>8</sup>

This setup captures the reality that even people who are totally unprepared for retirement in terms of their lack of savings will still need to decide when to retire. By assumption, the retirement choice of the hand-to-mouth consumer is the most desirable retirement choice under the constraint that nothing is saved. It is the best he can do given that he has failed to save. Although it is possible to imagine that such consumers also make poor decisions along the retirement dimension, we want to study the case where they are cognizant of the trade-off between leisure and higher benefits. In this way, like the LCPI consumer, they take full advantage of the rewards of the social security program. Indeed, it seems intuitive that, in reality, those who save the least may pay the most attention to the social security options available to them when they are on the verge of retirement.<sup>9</sup>

As in Hurst and Willen (2007), we do not take a stand on why hand-to-mouth consumers behave as they do. However, implicitly we are assuming that hand-to-mouth behavior is *not* the result of some underlying optimization problem. Instead, this behavior is suboptimal, and this creates a possible welfare improving role for social security.

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<sup>8</sup>Note that the hand-to-mouth consumer just makes a single choice, unlike the LCPI consumer who solves a dynamic problem.

<sup>9</sup>Problem (8)-(10) is time consistent.

## 2.3 Welfare Criterion

Regardless of the consumer type, the true lifetime utility function used for welfare considerations is

$$\int_0^T \exp[-\rho t] [\sigma \ln c(t) + (1 - \sigma) \ln l_-] dt + \int_T^{\bar{T}} \exp[-\rho t] \sigma \ln c(t) dt. \quad (11)$$

The LCPI consumer chooses consumption, saving, and retirement to maximize this function, while the hand-to-mouth consumer does not maximize this function. The hand-to-mouth consumer may potentially benefit from a mandatory pension program, even if contributions earn a below market internal rate of return. Such a paternalistic view of welfare is common (and necessary) in studies which attempt to rationalize social security in dynamically efficient economies by appealing to myopia.

The standard result in mixed economies such as the one we are studying, but with exogenous retirement, is that it is *always* optimal to have some social security program (Feldstein (1985)). This is because the hand-to-mouth consumers would starve during retirement without a social security program. However, this is not necessarily the case if retirement is endogenous. The hand-to-mouth consumer can choose to work longer (provided he is not fully disabled) to avoid destitution, and whether or not working longer is welfare improving is a quantitative question.

## 3 Aggregation and General Equilibrium

Suppose a fraction of consumers  $\Lambda \in (0, 1)$  are hand-to-mouth and the rest are LCPI. At each instant in time a cohort of size  $N$  is born. Every cohort bears the same proportion of consumer types. The total supply of labor  $L$  at any point in time is

$$L = \int_0^{T_H^*} \Lambda N(1 - l_-) e(t) dt + \int_0^{T_P^*} (1 - \Lambda) N(1 - l_-) e(t) dt, \quad (12)$$

where the subscripts  $H$  and  $P$  stand for “hand-to-mouth” and “permanent-income.” Unless by chance in the calibration,  $T_H^*$  and  $T_P^*$  will not be the same, and hence the supply of aggregate labor depends on the proportion  $\Lambda$ .

Aggregate demand for capital  $K$  at any point in time is

$$K = \int_0^{\bar{T}} (1 - \Lambda) N k(t) dt. \quad (13)$$

Output is Cobb-Douglas

$$Y = K^\alpha L^{1-\alpha}, \quad (14)$$

where  $\alpha$  is capital’s share. Factors of production are priced competitively

$$r = Y_K - \delta = \alpha \frac{Y}{K} - \delta, \quad (15)$$

$$w = Y_L = (1 - \alpha) \frac{Y}{L}, \quad (16)$$

where  $\delta$  is the rate of depreciation of capital.

The social security system operates a pay-as-you-go pension program, so the budget must balance at each instant in time. Because the two groups of consumers may choose to retire at different ages, two different benefit levels may prevail in equilibrium,  $b(T_H^*)$  and  $b(T_P^*)$ . The social security budget will balance if we impose the restriction

$$\theta wL = b(T_H^*) \times \Lambda N(\bar{T} - T_H^*) + b(T_P^*) \times (1 - \Lambda)N(\bar{T} - T_P^*), \quad (17)$$

where the left side of (17) is total tax revenue collected at an instant in time and the right side is total benefits paid to retirees at the same instant in time. Note that  $\Lambda N(\bar{T} - T_H^*)$  is the total number of hand-to-mouth retirees and  $(1 - \Lambda)N(\bar{T} - T_P^*)$  is the total number of LCPI retirees. All hand-to-mouth retirees receive the same social security annuity, which depends on the age that this group chooses to retire. Likewise all LCPI retirees receive the same annuity, which depends on their retirement age.

Combine (17) and (5) and solve for  $z_0$

$$z_0 = \frac{\theta wL - \frac{z_1 \Lambda N(\bar{T} - T_H^*)}{1 + \exp[z_2 - z_3 \times T_H^*]} - \frac{z_1 (1 - \Lambda) N(\bar{T} - T_P^*)}{1 + \exp[z_2 - z_3 \times T_P^*]}}{\Lambda N(\bar{T} - T_H^*) + (1 - \Lambda) N(\bar{T} - T_P^*)}. \quad (18)$$

Thus, we can exogenously set  $z_1$ ,  $z_2$ , and  $z_3$  and then let  $z_0$  be determined in general equilibrium to ensure the budget balances.

A **stationary competitive equilibrium** summarizes the model and is defined by

- consumption rules that obey (6) for the LCPI consumers and (9)-(10) for the hand-to-mouth consumers given factor prices  $r$  and  $w$ , tax rate  $\theta$ , and social security parameters  $z_0$ ,  $z_1$ ,  $z_2$ , and  $z_3$ ;
- endogenous retirement dates that maximize utility given factor prices  $r$  and  $w$ , tax rate  $\theta$ , and social security parameters  $z_0$ ,  $z_1$ ,  $z_2$ , and  $z_3$ ;
- factor prices  $r$  and  $w$  that are constant and satisfy (15) and (16) for a given aggregate capital stock  $K$  and labor  $L$ ;
- aggregate factor quantities  $K$  and  $L$  that are constant and satisfy (12) and (13) for given LCPI savings demand  $k(t)$  and retirement choices among both groups; and,
- a social security benefit rule  $b(T)$  where the scale parameter  $z_0$  satisfies the balanced budget constraint (18), for a given wage rate and for given choices of retirement dates.

If all of these conditions are simultaneously met, we have a stationary competitive equilibrium. Finding the equilibrium is a quantitative matter. See the Appendix for details on the computational procedure.

## 4 Quantitative Analysis

We begin this section by making a few simplifying assumptions. Next, we discuss the targets in the US data that we want our model to match. Then we calibrate our model and discuss the performance of the model in replicating the data targets. And finally we use the calibrated model to study the welfare effects of changes in the social security tax rate.

## 4.1 Simplifying Assumptions

Let us consider a stylized case for the labor efficiency profile

$$e(t) = e_{\max} \exp[-\mu t], \quad e_{\max}, \mu \in \mathbb{R}^+, \quad (19)$$

where  $e_{\max}$  is the maximum efficiency of labor and  $\mu$  is the rate of (labor efficiency) depreciation. This specification captures the notion that the individual grows less productive with age and hence there will be a motive to retire at some point.<sup>10</sup> In the real world, of course, efficiency profiles are hump shaped over the life cycle with a peak near the middle of the working period. However, the upside of the efficiency profile is not important for the retirement choice. It is the downside (backside) of the efficiency profile that leads the individual to ultimately leave work since his labor effort generates smaller and smaller returns with age. This simplifying assumption would be problematic if we were interested in the shape of the leisure profile over the working period, but this is not the purpose of our study as we have already assumed that leisure is fixed over the working period in order to focus on the retirement decision.

For the LCPI consumer, (6) can be written compactly as

$$c_T(t) = X \exp[(r - \rho)t], \quad (20)$$

where

$$X \equiv \frac{\int_0^T (1 - \theta)(1 - l_-) w e_{\max} \exp[-(\mu + r)s] ds + \int_T^{\bar{T}} b(T) \exp[-rs] ds}{\int_0^{\bar{T}} \exp[-\rho s] ds}, \quad (21)$$

which is then inserted into (7) to find the optimal retirement date

$$T_P^* = \arg \max \left[ \begin{array}{l} \int_0^T \exp[-\rho t] (1 - \sigma) \ln l_- dt + \sigma \int_0^{\bar{T}} \exp[-\rho t] \ln X dt \\ + \sigma \int_0^{\bar{T}} \exp[-\rho t] (r - \rho) t dt \end{array} \right]. \quad (22)$$

Note that the last term on the right of (22) is independent of  $T$ , so

$$T_P^* = \arg \max \left[ \int_0^T \exp[-\rho t] (1 - \sigma) \ln l_- dt + \sigma \int_0^{\bar{T}} \exp[-\rho t] \ln X dt \right]. \quad (23)$$

Meanwhile, using (19) the optimal retirement date for the hand-to-mouth consumer is

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<sup>10</sup>Another simple way to introduce a motive for retirement is found in Cremer, Lozachmeur, and Pestieau (2004). They modify the period utility function directly so that disutility from work gets larger with age. Also see Gustman and Steinmeier (2005).

found by inserting (9) and (10) into (8), and then integrating by parts

$$T_H^* = \arg \max \left\{ \begin{aligned} & \left[ \sigma \ln ((1 - \theta)(1 - l_-)e_{\max} w) - \frac{\sigma \mu}{\rho} + (1 - \sigma) \ln l_- \right] \times \int_0^T \exp[-\rho t] dt \\ & + \frac{\sigma \mu}{\rho} T \exp[-\rho T] + \sigma \ln b(T) \int_T^{\bar{T}} \exp[-\rho t] dt \end{aligned} \right\}. \quad (24)$$

## 4.2 Targets in the US Data

There are a number of targets that are relevant to the study of the optimal provision of social security.

- **Target 1.** A capital-output ratio of 3.0 is a standard target in the calibration of life-cycle macro models. We want our model to generate the right level of aggregate capital accumulation since the optimal size of social security will depend on the level of private savings.
- **Target 2.** An interest rate of 3.5% is also a standard target in life-cycle macro models and we adopt it as well. It is important to have a realistic interest rate because this parameter affects the incentive to save.
- **Target 3.** We use workers' expected date of retirement from the 2009 Retirement Confidence Survey (RCS) of American households as the targets for retirement age. Our use of "expected" rather than "realized" retirement age is because a significant share of workers tend to leave work earlier than expected for reasons relating to health and disability, which are concepts that are not modeled here. Hence, we want to calibrate the model to retirement ages that would prevail in the absence of such surprise shocks to focus on myopia (undersaving) as a rationale for social security. We treat the median planned retirement age of 65 as our target. The RCS also reports that 20% of Americans plan to work until at least 70, so we also want our model to generate "late" retirement for around 20% of the model population.
- **Target 4.** A large literature in macroeconomics documents a discrete drop in aggregate life-cycle consumption profiles at retirement. There is little disagreement about the existence of the drop, but the magnitude varies from study to study. We take 10% to 20% to be a reasonable target range. This target is important because it reflects the level of preparedness for retirement and also the replacement rates in the social security program. LCPI consumers do not experience a drop, but the hand-to-mouth consumers experience a drop that depends on the size of the social security program, their efficiency endowment at the date of retirement, and their chosen retirement age.
- **Target 5.** In the US social security system, the level of benefits received depends directly on the age at which benefits commence. Those who initiate benefits the earliest get the smallest annuity, and those who wait the longest get the largest annuity. As

of 2008, those who retire at the earliest possible date (age 62) receive a social security annuity that is permanently about 25% smaller than those who retire at the “normal” retirement age of 66. Those who retire at the latest possible date (age 70) receive about 35% more than those who retire at the normal age.<sup>11</sup> There is considerable cross-country evidence that this feature, which is common to a number of social security programs in the OECD, is indeed a strong predictor of labor force participation at older ages (Gruber and Wise (1997, 1998)). We target the ratio of the annuity at age 70 to the annuity at age 62, which is 1.8 in the US. Matching this target ensures that the consumers in the model face realistic retirement incentives. While this target may seem more like a parameter that can just be chosen at will, recall from (18) that  $z_0$  is determined in equilibrium and hence the ratio of benefits at 70 to benefits at 62 is also determined in equilibrium.

### 4.3 Calibration and Performance of the Model

Each of the above targets is an endogenous variable in our model. Thus, after setting observable parameters to standard values, our task is to select the remaining free parameters to replicate the targets as closely as possible.

We start with the observables. We assume that model time  $t = 0$  corresponds to age 25, and hence the lifespan  $\bar{T} = 55$  implies death at age 80. This is close to the average lifespan of individuals in the US. Hence,  $T + 25$  is analogous to the age of retirement in the real world. In what follows, when we say “retirement age” we are referring to  $T + 25$ . We normalize  $e_{\max} = 1$  and we set the rate of human capital depreciation to  $\mu = 1\%$ . This aligns closely with the rate of decline in the estimated life-cycle earnings profile of a typical US consumer (Gourinchas and Parker (2002)).<sup>12</sup> The social security tax rate is set to  $\theta = 10.6\%$ , which is the combined employer plus employee (OASI) social security tax rate in the US. Note that we do not include the additional tax that funds the disability program because we do not model disability. If there are 16 hours per day in non-sleep time (to be divided between work and leisure), then a worker who spends 40 hours per week on the job spends 36% of his non-sleep time at work. This suggests that  $l_- = 64\%$  is a reasonable assumption.<sup>13</sup> Retirement then provides a 50% increase in leisure time (from 64% to 100% of non-sleep time). Capital’s share  $\alpha = 35\%$  and the depreciation rate for capital is  $\delta = 8\%$ , which are also standard assumptions. Cohort size is normalized to  $N = 1$ . Following Hurst and Willen (2007) and others, 20% of the population live hand-to-mouth ( $\Lambda = 20\%$ ). This matches empirical estimates of the fraction of the population who never save. By assuming that these people fail to solve the LCPI optimization problem, rather than because they merely have a high discount rate, we bias the model in favor of finding a welfare improving role for social security.

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<sup>11</sup>Retiring after the age of 70 does not generate any additional increases in the social security annuity.

<sup>12</sup>The Gourinchas and Parker (2002) estimate of earnings over the life cycle is hump shaped, but as explained above, it is the downside of the hump that drives the retirement decision. So what matters for our calibration is that our efficiency profile declines at the same rate as the downside of their earnings profile. (Note that efficiency units and earnings share the same slope over the life cycle in our model).

<sup>13</sup>If in addition to sleep time we also take out time spent eating and commuting to work, then a 40 hour work week would imply a lower value for  $l_-$  (such as 55% in Ortiz (2009)).

The five remaining free parameters are the preference parameters  $\rho$  and  $\sigma$ , and the parameters from the logistic social security rule  $z_1$ ,  $z_2$ , and  $z_3$ . We set  $\rho = 3.5\%$ , which is a reasonable value in the context of standard macroeconomic models and we set  $\sigma = 38\%$ , which is also reasonable (e.g., Heijdra and Ligthart (2006) use a value of  $\sigma = 32\%$ ). The retirement ages of the consumers are sensitive to  $\sigma$ . We don't have much flexibility to deviate from this value and still produce realistic retirement choices. Finally, we use  $z_1 = 0.06$ ,  $z_2 = 33$ , and  $z_3 = 0.83$ . Together with the observable parameters from above, these five values for the free parameters allow the model to provide a decent fit to the targets.

The capital-output ratio in the model is 2.94 and the interest rate is 3.9%, which provides a decent fit to targets 1 and 2. The LCPI consumers retire at age 66 and the hand-to-mouth consumers retire at 69.3. Thus, because there are more LCPI consumers, the median retirement age in the model of 66 is close to the median retirement age in the RCS data. The late retirement among the hand-to-mouth consumers is also a good fit to the RCS data, both in terms of the age and the fraction of the population who retire late. The hand-to-mouth consumers experience a large drop in consumption at retirement equal to 47.8%. Because these consumers are 20% of the population, and because the LCPI consumers do not experience a drop, the model generates an aggregate (weighted average) drop equal to 10%. This falls in the lower end of the target range. Finally, the ratio of the social security annuity conditional on collection at age 70 to the annuity at age 62 is 1.74 in the model, which closely matches the US social security program.

Our stylized model is performing fairly well along some dimensions that are important for our research question. Broadly speaking, the model replicates targets that correspond to saving incentives and choices, retirement incentives and choices, and preparedness for retirement in general. The model appears to be a reasonable instrument for studying the welfare effects of changes in the social security tax rate.

Figure 1 is the logistic function from (5), where  $z_1$ ,  $z_2$ , and  $z_3$  are parameterized as mentioned above and  $z_0 = 0.067$  is determined in equilibrium. The graph shows the annuity value of social security benefits as a function of retirement age. Before the age of 62, individuals cannot receive social security benefits in the US, yet in our model the individual can begin collecting benefits at any age. But the poor fit prior to 62 is irrelevant because the incentive structure causes people in the model to retire after age 62 anyway. Thus we do not need to explicitly impose a constraint for the minimum eligibility age.

## 4.4 Welfare Analysis

To study the welfare effects on lifetime utility of cutting the social security tax rate from a high value to a lower value, we define the compensating variation  $CV$  as the percentage increase in consumption in each period that an individual in the equilibrium with the high tax would need to provide the same utility as in the equilibrium with the low tax. Let  $dU$  stand for the lifetime utility in the low tax equilibrium minus lifetime utility in the high tax equilibrium. Using the definition of lifetime utility (11), we can solve analytically for the compensating variation

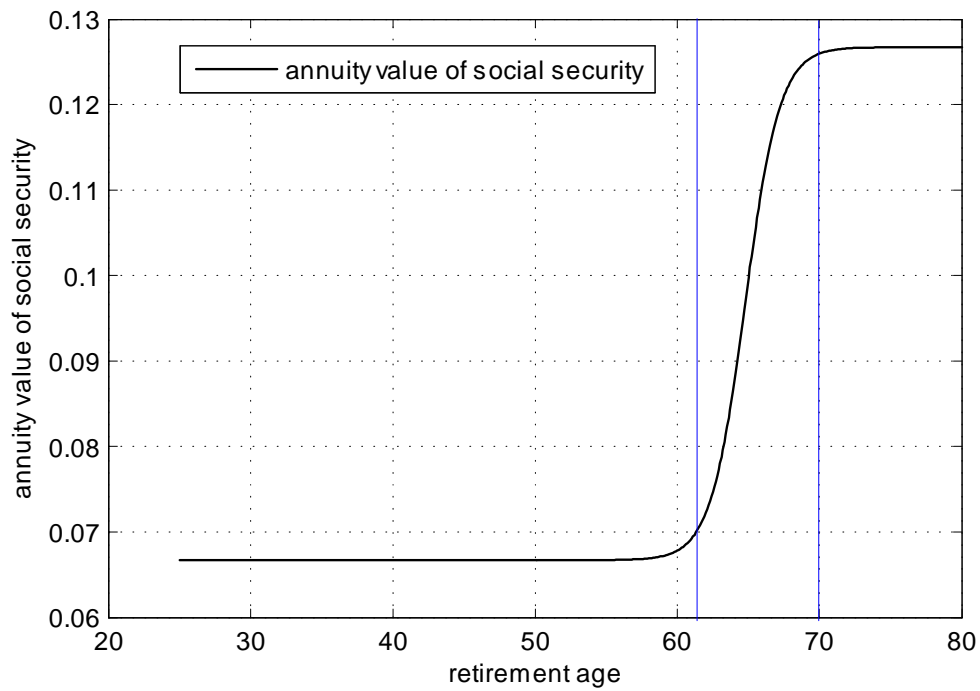


Figure 1: Logistic Approximation of Social Security Benefit Rule as a Function of Retirement Age. The vertical lines are purely for reference, to draw attention to the early (age 62) and late (age 70) retirement dates in the US program.

$$CV = \exp \left[ \frac{dU}{\sigma \int_0^{\bar{T}} \exp[-\rho t] dt} \right] - 1. \quad (25)$$

Figure 2 graphs the compensating variation for both LCPI and hand-to-mouth consumers, as a function of the size of the reduction in the social security tax rate (measured in percentage points). The compensating variations are similar in shape and magnitude for the two consumers, and the magnitudes are clearly significant. For example, a 1.6-point reduction in the social security tax rate from 10.6% to 9.0% generates compensating variations of 1.4% for the LCPI consumers and 1.2% for the hand-to-mouth consumers. We can put the magnitude of these utility gains into context by noting that the compensating variation of eliminating idiosyncratic shocks to wage income, which is a major source of macroeconomic research, is around 1-2% (Vidangos (2008)). Figure 3 shows the retirement behavior behind the tax cuts. Both consumers respond to a tax cut by delaying retirement, but the hand-to-mouth consumer is more responsive.<sup>14</sup>

There are a number of complex effects behind these welfare results. Let us start with the macroeconomic effects. Moving from a social security tax rate of 10.6% to 9.0% causes the aggregate capital stock to increase by 2% because the LCPI individuals increase their saving. The labor supply increases by 1.5% due to delayed retirement from both consumer types. The growth in these factors leads to a 1.7% increase in aggregate output. Output net of depreciation,  $Y - \delta K$ , goes up by 1.6%. Since households are the owners of the factors of production, they receive larger factor payments at the aggregate level. These aggregate changes are reflected in a decrease in the interest rate and in increase in the wage rate.

At the micro level, it is no longer the case that the internal rate of return in social security equals the rate of growth of the economy (Feldstein (1985)). While this is true at the aggregate level, the two groups of consumers retire at different points in time, which affects the internal rate of return. The group that retires later bears a larger portion of the tax burden, but on the other hand this groups receives a larger benefit annuity. In theory, the group with the higher internal rate of return is ambiguous. Also, the internal rate of return experienced by the two groups is no longer independent of the magnitude of the social security tax because the dates of retirement are endogenous and are therefore sensitive to the tax rate. In sum, although the LCPI consumers exploit the hand-to-mouth consumers by retiring earlier (and therefore paying less taxes), the hand-to-mouth individuals recover some of these losses through larger benefits. In the baseline equilibrium, the LCPI individuals do not exploit the hand-to-mouth individuals by enough to favor a social security program (which would only occur if their internal rate of return exceeds the market interest rate), and so they are better off with a reduction in the tax rate. Indeed, LCPI consumption is

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<sup>14</sup>Cremer and Pestieau (2000) summarize a literature which illustrates how, under majority voting, a social security program can naturally grow bigger than is socially desirable. Basically, everyone who experiences an internal rate of return on social security that beats the market, from the perspective of where they are currently standing in the life cycle, will vote for a large program. Those near or at retirement, together with low income individuals match this description and they form a majority. See Cremer, De Donder, Maldonado, and Pestieau (2007) for more on voting.

everywhere higher across the life cycle in the equilibrium with lower taxes, although the slope of the consumption function is less steep due to the decrease in the interest rate (recall the standard Euler equation).

Though the hand-to-mouth consumers would always prefer that the internal rate of return be as high as possible other things equal, the relationship between the internal rate of return and the market rate of interest is less relevant. We no longer have the knife-edge relationship that we have with LCPI consumers. Hand-to-mouth consumers, in theory, can potentially benefit from a program with a below market internal rate, though our calculations show a reduction in the tax rate helps them too. The lower tax allows the hand-to-mouth individual to enjoy higher consumption during the working period due to higher disposable wage income, because the tax rate is lower and because the wage rate is higher. However, even though he works longer, the lower tax rate causes the annuity value of his benefits to be lower than in the baseline equilibrium. Thus, the hand-to-mouth consumer enjoys higher consumption during the working period but suffers lower consumption during retirement and a longer working period. Quantitatively, the gains outweigh the losses and his lifetime utility is higher with lower taxes.

If a small decrease in the tax rate generates welfare gains for both consumer types, then what is the optimal tax rate? We have found many parameterizations of the model where it is actually optimal to have no social security program at all, but we don't want to emphasize this result. The problem is that the hand-to-mouth individual has to work up until the date of death if there is no social security program, so a serious analysis of eliminating the social security program would need to account for disability. Because the agent in our model, by assumption, never becomes fully disabled, clearly our model should not be used to study the consequences of such extreme changes in the social security program. This is why we have limited our welfare analysis to small changes in the tax rate. Small changes in the tax generate small changes in retirement choices. The LCPI consumer delays retirement by just a few months and the hand-to-mouth consumer delays retirement by approximately 3 years when we drop the tax from 10.6% to 9.0%.<sup>15</sup> Ignoring disability seems less problematic when retirement responses are at this order of magnitude. To consider bigger reductions in the tax rate, we must confront the possibility that some people simply cannot respond by working longer. We leave this on the future agenda.

## 5 Synchronization of Retirement with Benefits

We need to mention a caveat. So far, we have made the simplifying assumption that initiation of benefits is synchronized with retirement from the labor force. This is not necessarily the case in reality, since some individuals may retire early but wait until later to begin collecting benefits. This assumption is totally harmless for the hand-to-mouth consumer. He could

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<sup>15</sup>Note that the predictions of our general equilibrium model are consistent with the vast empirical evidence suggesting that growth in the generosity of social security programs across the world may be responsible for part of the tendency toward earlier retirement. Yet, the LCPI retirement age doesn't change much because we have simply scaled down the size of the program, and as in Samwick (1998), the LCPI retirement choice has more to do with the benefit formula  $b(T)$  than the magnitude of the tax. A change in the benefit rule  $b(T)$ , such as setting  $z_1 = 0$ , can have a very large effect on the retirement choices of both consumers in the model.

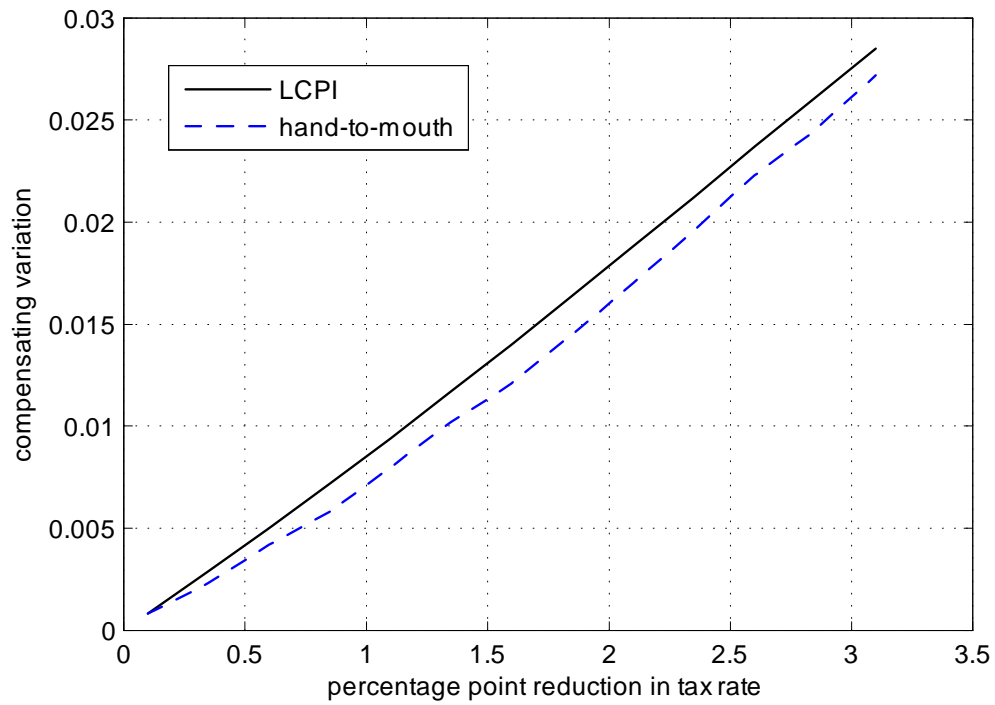


Figure 2: Compensating Variation as a Function of the Reduction in the Social Security Tax Rate. This is the percentage increase in consumption in each period that is required to bring the individual's lifetime utility in the baseline equilibrium (with the current tax rate) up to the level in a different equilibrium with a lower tax rate.

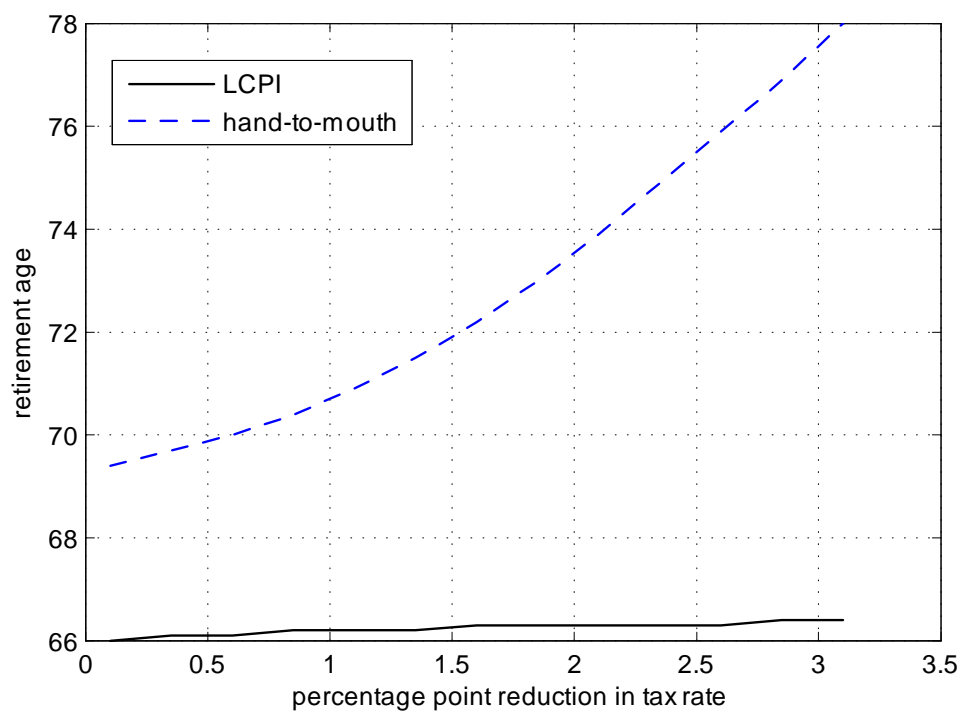


Figure 3: Retirement Age as a Function of the Reduction in the Social Security Tax Rate.

not delay collection of benefits because he has no savings to support consumption spending between retirement and benefit initiation. But the LCPI consumers may optimally choose early retirement from the labor force followed by late initiation of benefits.<sup>16</sup> Until now, we have made no attempt to de-couple the retirement choice from the benefit initiation choice. We have followed Diamond and Gruber (1997) and treated them as a single, combined choice in the calculations above.<sup>17</sup>

Suppose the age of retirement  $T$  for an LCPI consumer potentially differs from the age at which benefits are initiated  $T_0$ , where  $T_0 \geq T$ .<sup>18</sup> Benefits would depend on  $T_0$  rather than  $T$ . Standing at any given date of retirement  $T$ , looking forward the present value of social security benefits as a function of the initiation date  $T_0$  is

$$PV(T_0|T) = \int_{T_0}^{\bar{T}} \exp[-r(t-T)] b(T_0) dt, \text{ for all } T_0 \in [T, \bar{T}]. \quad (26)$$

Because the initiation date and the retirement date are now separate choices, the LCPI consumer will maximize lifetime utility by choosing to initiate benefits in a way that maximizes his discounted wealth, conditional on a given retirement date

$$T_0^*(T) = \arg \max_{T_0 \geq T} PV(T_0|T), \text{ for all } T. \quad (27)$$

If the function  $PV(T_0|T)$  is always decreasing in  $T_0$ , for all relevant retirement dates  $T$  (by “relevant” we mean between the ages of 62 and 70, where the incentive structure is relevant), then it would be the case that a rational individual would never postpone collection of benefits beyond any date of retirement. And, this would then imply that we are justified in assuming that initiation of benefits and retirement from the labor force are a single, synchronized choice, since it would never be optimal to do otherwise. Note that  $PV(T_0|T)$  is determined in general equilibrium because it is a function of  $r$  and  $b(T_0)$ , which are endogenous.

However, for the calibration that we have been using above, plotting (26) shows that  $PV(T_0|T)$  can rise in  $T_0$  for some  $T$ , meaning that  $T_0^*(T) > T$  for some  $T$ . In these cases, it would be optimal to delay initiation of benefits beyond retirement, conditional on retirement at those particular dates. We emphasize that this does not necessarily mean the optimal overall plan is to delay initiation, since there could be other retirement dates where  $T_0^*(T) = T$  which dominate, in a lifetime utility sense, those retirement dates for which delayed initiation is optimal. Figure 4 shows the graph of  $PV(T_0|T)$ , where retirement is fixed at age 62. Here we see that, conditional on that retirement age, it would be optimal to postpone the collection of benefits until age 66.<sup>19</sup> On the other hand, Figure 5 shows the same graph

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<sup>16</sup>The optimal timing of benefit collection is the subject of much debate in the household finance literature, though it is emphasized far less in the typical macro models with social security (Coile, Diamond, Gruber, and Jousten (2002)).

<sup>17</sup>This simplified modeling approach could potentially be justified on the grounds that, in reality, delayed claiming is “far less prevalent than theory predicts” (Coile, Diamond, Gruber, and Jousten (2002)). Also Casamatta, Cremer, and Pestieau (2005) argue that a bias towards early retirement in the design of social security is both desirable in a utilitarian sense and supportable by majority voting. Also see Cremer, Lozachmeur, and Pestieau (2004).

<sup>18</sup>Collecting benefits before retirement is possible in reality but is not considered here. See Ortiz (2009).

<sup>19</sup>The shape of this graph is consistent with Coile, Diamond, Gruber, and Jousten’s (2002) calculation that anything less than a 6% discount rate argues for delayed claiming (our discount rate is much lower than 6%).

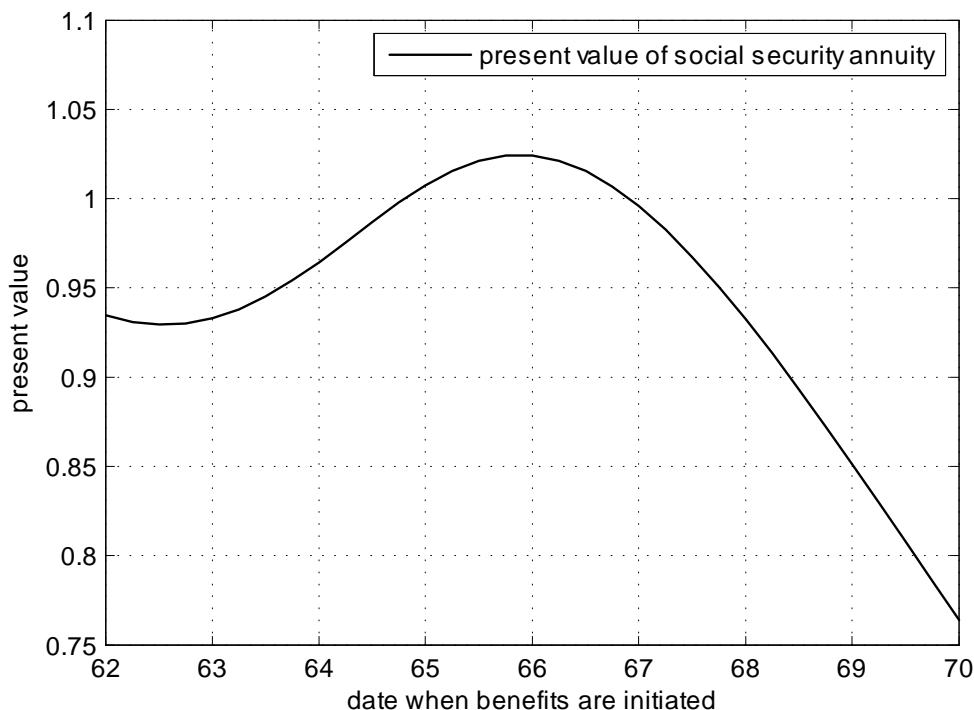


Figure 4: Present Value of Social Security Annuity, Conditional on Retirement at Age 62.

but where retirement is fixed at age 66. Conditional on retirement at this later date, it would not be optimal to postpone the collection of benefits. Thus, at the baseline equilibrium, an LCPI consumer would need to compare lifetime utility from retiring at 62 and initiating benefits at 66 on the one hand, against retiring at 66 and initiating benefits at 66 on the other hand. (Retiring at 62 and collecting benefits at 62 is a strictly dominated strategy). All other retirement dates would likewise be considered by the LCPI consumer.

But we think this issue does not affect the main quantitative results of the paper. For example, suppose we consider a logistic benefit rule with a slightly less steep jump (i.e., slightly smaller rewards for delaying initiation of benefits). That is, suppose we set  $z_2 = 16$  rather than 33 as in the baseline above, and  $z_3 = 0.40$  rather than 0.83. Doing this will, of course, generate a totally different equilibrium with different factor prices and household choices. Figure 6 plots the baseline benefit rule (the solid line) versus the alternative benefit rule (dashed line). The figure also illustrates how the two groups of consumers change their chosen age of retirement. In moving from the baseline to the alternative equilibrium, the LCPI consumers choose to retire 2 years earlier at age 64 whereas the hand-to-mouth consumers retire about 1.5 years later. These retirement dates at the alternative equilibrium are still in-line (or perhaps even slightly better in-line) with the RCS evidence targeted above.

The alternative benefit rule from Figure 6 still bears a pronounced logistic shape with significant rewards for waiting to collect benefits, but now it is strictly the case that  $PV(T_0|T)$

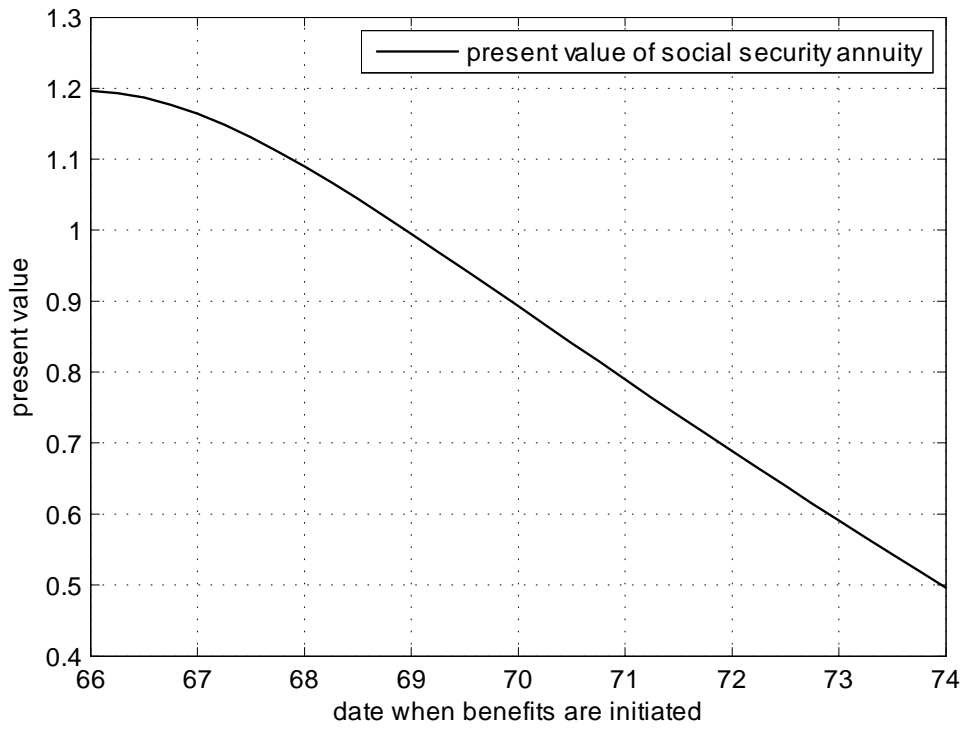


Figure 5: Present Value of Social Security Annuity, Conditional on Retirement at Age 66.

is a decreasing function of  $T_0$ , for *all* relevant retirement dates  $T$  (i.e., unlike Figure 4). This means that at this alternative equilibrium, there would never be any incentive to postpone collection of benefits beyond any relevant date of retirement, even though there is still a strong incentive to delay retirement itself due to the logistic benefit rule. That is, there is a strong incentive to delay retirement relative to the case where the benefit annuity does not depend on the initiation date (e.g., France’s system, until recently).<sup>20</sup>

Most importantly, if we recompute the compensating variation for the alternative equilibrium that comes from reducing the social security tax rate, we get almost identical numerical results. Figure 7 is a replica of Figure 2, but with the alternative parameterization of the social security benefit rule. Clearly, Figures 2 and 7 hardly differ in any material way in terms of the welfare improvements to both groups of consumers associated with a reduction in social security benefits. Yet, behind the welfare calculations the two equilibria are fundamentally different because it is never optimal to delay initiation of benefits for any retirement age in the alternative equilibrium. This seems to imply that our baseline assumption about the synchronization of retirement with collection of benefits, even if potentially suboptimal behavior for some particular retirement dates, is not the reason for the large welfare gains associated with small reductions in the social security tax rate.

Finally, in the Appendix we take a more detailed approach to demonstrating the robustness of our welfare results. We attempt to build the type of model that Coile, Diamond, Gruber, and Jousten (2002) have put on the research agenda:

*“A full analysis of the problem of delayed claiming would model jointly the retirement and the claiming decision.”*

We make progress along this front by augmenting our optimal control problem of endogenous retirement to also allow for delayed claiming beyond the date of retirement. The problem becomes a three-stage optimal control problem with two endogenous switch points, requiring an extra loop in the computational procedure. The agent now jointly optimizes along the consumption, retirement, and benefit collection dimensions.

At the baseline calibration from Section 4.3, we find that allowing the LCPI individual the option to delay initiation would ultimately not affect our welfare calculations. Indeed, he continues to choose age 66 as his retirement date *and* he will choose to begin collecting benefits right away at the same age. This also then means that consumption and saving are unchanged over the life cycle, leaving lifetime utility exactly the same as when delayed collection was not allowed. Next, we consider the equilibrium from Section 4.4 with the lower social security tax rate (9.0%). We find that the individual would choose to retire at age 66.3 and also begin collecting benefits at 66.3, which is precisely the same age and collection date that we get when we impose the synchronization constraint. As with the baseline equilibrium, behavior in this lower-tax equilibrium is not affected in any way by

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<sup>20</sup>The alternative equilibrium in Figure 6 may be particularly relevant for other industrialized countries. Gruber and Wise (1997) summarize the large NBER project “Social Security Programs and Retirement Around the World,” which is a compilation of 11 country-specific studies, and they conclude that postponing collection of benefits does not typically increase the present value of the social security benefit annuity. Thus, while many countries do provide incentives to compensate people for postponing collection of benefits, the incentives are generally not strong enough to make our baseline assumption about synchronization too problematic in the international context.

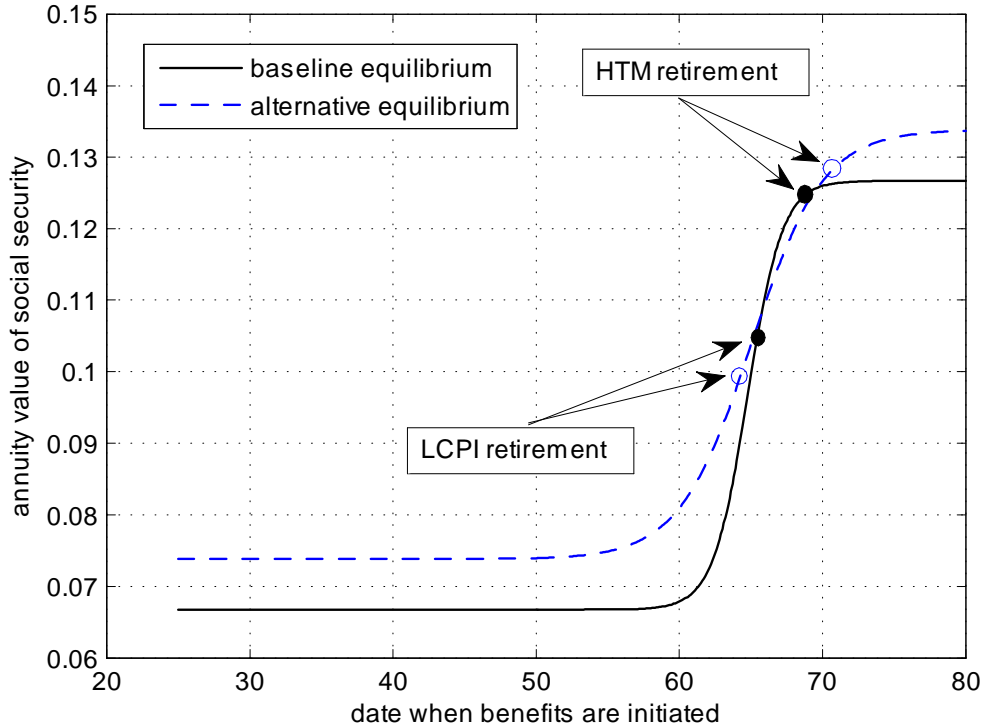


Figure 6: Annuity Value of Social Security as a Function of Collection Date: Baseline Benefit Rule Versus an Alternative Benefit Rule. The circles indicate the optimal retirement ages across the different general equilibria. The dark circles correspond to the baseline, and the light circles correspond to the alternative.

the synchronization constraint and so lifetime utility would be the same with or without the constraint. Putting all of this together, the compensating variation from a reduction in the social security tax rate will not vary at all from the calculation in Section 4.4. This does not mean that delayed collection of benefits is unimportant in general. Rather, it is unimportant at the calibrations of the model that we are considering because the LCPI consumer ultimately chooses to retire from the labor force at an age that would warrant any more delays in the collection of benefits. In sum, we are reasonably confident that our welfare calculations associated with lowering the social security tax rate are not a function of the baseline assumption that retirement and benefits are a single, synchronized choice.

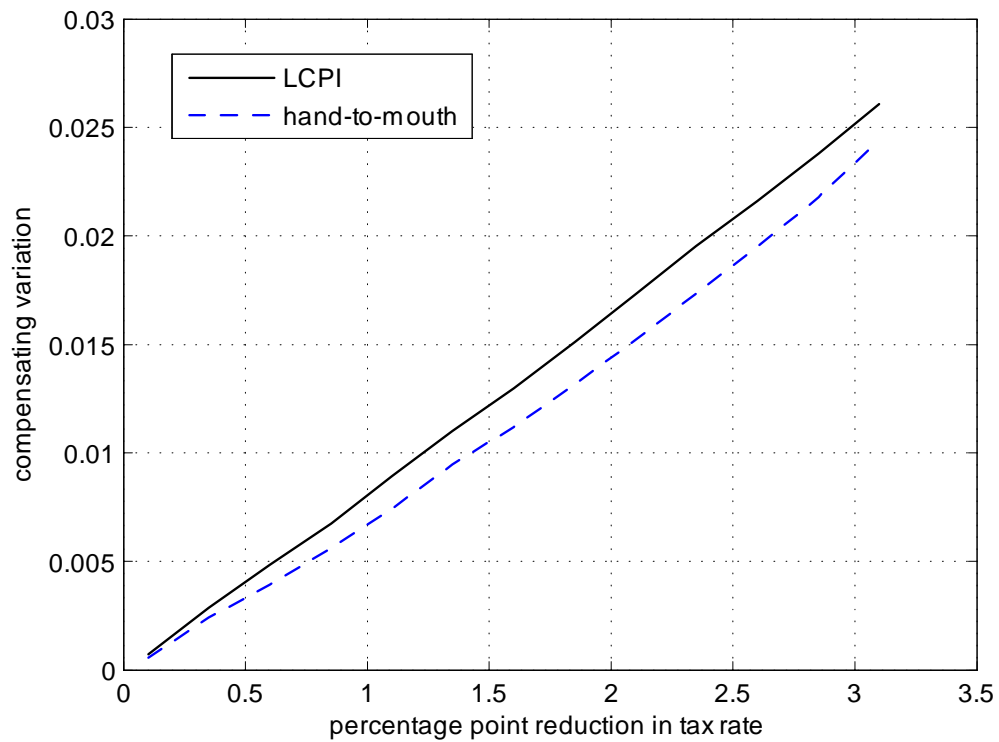


Figure 7: Compensating Variation from a Reduction in the Social Security Tax Rate. This is a replica of Figure 2, but with alternative parameterization of the logistic benefit rule.

## 6 Concluding Remarks and Direction for Future Work

We have built a life-cycle general equilibrium model that matches a number of important quantitative targets relating to saving and retirement. In the model, there are large welfare gains to both life-cycle and hand-to-mouth consumers associated with small reductions in the social security tax rate. Thus, in the context of the present model, the social security program is bigger than can be justified by the level of myopia present in the US, leaving other roles such as redistribution of income as a necessary part of the rationale for social security. But we take a modest interpretation of the contribution of our paper. Although we have made some progress toward the question at hand, we have really just made a start. Like any model, we have abstracted from reality along a number of dimensions, and some of these dimensions could turn out to be important. For example, while we have allowed for the possibility of postponing claiming beyond retirement, in actuality it can also work the other way too. Collection of benefits may occur before retirement, but with large tax penalties. Future work may enrich our model to include this feature.

The normative literature concerning the optimal design of social security programs has focused mostly on the optimal tax rate and the optimal degree of redistribution. Another interesting dimension is the optimal benefit rule as a function of age. Many countries in the OECD now have public pension programs that provide incentives for postponing retirement. While much has been said about the positive implications for labor force participation, there has been less focus on the normative question of whether such incentives are desirable. Cremer and Pestieau (2003) and Cremer, Lozachmeur, and Pestieau (2004) have pushed forward along this front in models with rational individuals where social security serves a redistribution objective. Their work could be extended to include myopia or some type of bounded rationality, and it would be interesting to study the welfare consequences of this institutional feature in such a setting, especially since a number of countries in the OECD have recently converted their “flat” benefit rule into one that increases with age. Our preliminary calculations suggest that there are many complex general equilibrium effects that make this a non-trivial research question for future work.

## 7 Appendix: More on Synchronization

Here we take a more direct approach to illustrating the robustness of our analysis to the synchronization constraint by relaxing it directly. Now the consumer's problem consists of choosing the optimal control variable  $c(t)$  and the optimal control parameters  $T$  and  $T_0$

$$\max_{\{c(t)\}, T, T_0} \int_0^T \exp[-\rho t] [\sigma \ln c(t) + (1 - \sigma) \ln l_-] dt + \int_T^{\bar{T}} \exp[-\rho t] \sigma \ln c(t) dt, \quad (\text{A1})$$

subject to

$$\frac{dk(t)}{dt} = rk(t) + (1 - \theta)(1 - l_-)e(t)w - c(t), \text{ for } t \in [0, T], \quad (\text{A2})$$

$$\frac{dk(t)}{dt} = rk(t) - c(t), \text{ for } t \in [T, T_0], \quad (\text{A3})$$

$$\frac{dk(t)}{dt} = rk(t) + b(T_0) - c(t), \text{ for } t \in [T_0, \bar{T}], \quad (\text{A4})$$

$$k(0) = k(\bar{T}) = 0, \quad (\text{A5})$$

$$b(T_0) = z_0 + \frac{z_1}{1 + \exp[z_2 - z_3 \times T_0]}, \quad z_0, z_1, z_2, z_3 \in \mathbb{R}^+, \quad (\text{A6})$$

$$T_0 \geq T. \quad (\text{A7})$$

This is a three-stage optimal control problem with two endogenous switch points  $T$  and  $T_0$  (we could have split the objective functional into three integrals, as we might expect from a three-stage problem, but the integrand is the same in the second and third phases so we can combine). We can decompose the control problem into an inner control subproblem of choosing  $c(t)$  optimally, conditional on  $T$  and  $T_0$ , and then an outer problem of jointly choosing  $T$  and  $T_0$  given the optimized value of  $c(t)$ .

Thus, for a fixed retirement date  $T$  and a fixed initiation date  $T_0$ , the solution to the control subproblem is

$$c_{T, T_0}(t) = X \exp[(r - \rho)t], \quad (\text{A8})$$

where

$$X \equiv \frac{\int_0^T (1 - \theta)(1 - l_-)w e_{\max} \exp[-(\mu + r)s] ds + \int_{T_0}^{\bar{T}} b(T_0) \exp[-rs] ds}{\int_0^{\bar{T}} \exp[-\rho s] ds}. \quad (\text{A9})$$

Then, the optimal retirement and initiation dates are chosen jointly according to (substitute (A8) into the objective functional in (A1) and drop terms unrelated to both  $T$  and  $T_0$ )

$$\max_{T, T_0} \left[ \int_0^T \exp[-\rho t] (1 - \sigma) \ln l_- dt + \sigma \int_0^{\bar{T}} \exp[-\rho t] \ln X dt \right], \text{ subject to } T_0 \geq T. \quad (\text{A10})$$

Note the maximand in (A10) is the value functional from the control subproblem (with terms unrelated to  $T$  and  $T_0$  omitted).

## 8 Appendix: Computational Procedure for the Stationary Competitive Equilibrium

After assigning values to all parameters, the following steps are used to find an equilibrium.

**Step 1.** Guess values for the aggregate capital stock  $K$  and retirement dates  $T_P$  and  $T_H$  and call these three guesses the vector  $\mathbf{v}_{\text{guess}}$ .

**Step 2.** Based on  $\mathbf{v}_{\text{guess}}$ , compute aggregate labor  $L$ , aggregate output  $Y$ , factor prices  $r$  and  $w$ , and the social security scale parameter  $z_0$ .

**Step 3.** Using  $r$ ,  $w$ , and  $z_0$  compute the LCPI choice of consumption, saving, and retirement and the hand-to-mouth choice of retirement.

**Step 4.** Aggregate the LCPI saving profile to find  $K$  and put this value, together with the retirement choices from the previous step in a vector called  $\mathbf{v}_{\text{feedback}}$ .

**Step 5.** Let  $\mathbf{v} \equiv \mathbf{v}_{\text{guess}} - \mathbf{v}_{\text{feedback}}$ . Iterate on  $\mathbf{v}_{\text{guess}}$  until  $\mathbf{v} \cdot \mathbf{v} < 0.01$ .

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